SOLUTION FOR A PLANE SHOCK WAVE MOVING THROUGH A LIGHTLY CURVED INTERFACE OF TWO MEDIA

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PMM Vol.29, № 4, 1965, pp. 658-666

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(Received February 2, 1965)

An investigation of the development of small perturbations of gas dynamic parameters behind the front of a plane shock wave moving from a highly rarefied (weightless) gas into a dense (perfect) gas is presented. The interface of the two gases is assumed to be lightly curved. An analytical solution is given for a simplified case of Richtmyer's problem [1] which was solved by him by numerical methods. Several papers dealt with the stability of plane shock waves propagating in a homogeneous medium, notably those by Freeman [2], D'iakov [3], Zaidel' [4], and also the substantial work of Iordanskii [5]. These investigations prove that a plane shock wave is stable. The solution of problems considered in this work follows the method proposed in [4] which leads to a system of hyperbolic equations with conditions along moving boundaries.

An analytic solution of this problem in linear approximation is given; the amplitude of the wave front distortions, and the propagation of such distortions in respect of time are analyzed. Proof is given that amplitude changes of the front of a weak shock wave are independent of boundary conditions; the law is the same for a wave from a lightly curved piston, as for a wave penetrating through a lightly curved interface (3.22)

For a strong shock wave the amplitude of distortion of its front is similarly independent of the boundary conditions (3.15) and (3.16).

However, the amplitudes of distortion of the front of a shock wave, and of a strong shock wave are notably different; in the first case the amplitude is proportional to $s^{-1/2}$, while in the second it is proportional to approximately $\sim s^{-3/2}$.

1. Statement of problem. Let it be assumed that the interface of two spaces, one filled with an undisturbed gas, and the other with a weightless gas, is in the plane YZ . The term weightless gas, in this context, means a gas of zero density and infinite velocity of sound, so that compressibility effects in it are absent. At time $t = 0$ a pressure, constant in time, is created in the weightless gas which generates a plane shock wave, the front of which at moment $t = 0$ takes the form of the interface. A shock wave at constant velocity D will move through the gas. As one of the spaces is filled with a weightless gas, there is no reflected wave. Let ρ_0 and c_0

denote the initial density and the local velocity of sound, and ρ and c the same parameters behind the shock wave front.

For simplicity of calculations we assume the dense gas to be a perfect gas with the isentropic index γ. Denoting the velocity if the undisturbed interface by U and the velocity of the wave in relation to the interface by V, we have $D = V + U$. Introducing parameter $\delta = 1/M_0^2$, where $M_0 = D/c_0$ is the Mach number, we obtain the known relationship

$$\sigma = \frac{\rho}{\rho_0} = \frac{h}{1+(h-1)\delta}, \quad V = \frac{D}{\sigma}, \quad \beta^2 = \frac{V^2}{c^2} = \frac{1+(h-1)\delta}{(h+1)-\delta}, \quad h = \frac{\gamma+1}{\gamma-1}$$

Having thus obtained the solution of the problem for undisturbed conditions, we shall analyze in linear approximation the propagation of the shock wave at its entry into the lightly curved interface of the perfect and the weightless gases. Without limiting the generality, we shall assume that the interface is slightly curved in one direction only, its form being determined by $\varepsilon(Y)$. The explicit form of function $\varepsilon(Y)$ will be established later. We shall use the same symbols as in [4], and a system of coordinates in which the interface is static.

The following linearized equations apply to perturbations of pressure p' and velocity components v_x' and v_y' in the region where $0 < X < Vt$ (1.1)

$$\frac{\partial p'}{\partial t} + \rho c^2 \left(\frac{\partial v_x'}{\partial X} + \frac{\partial v_y'}{\partial Y}\right) = 0, \quad \frac{\partial v_x'}{\partial t} + \frac{1}{\rho}\frac{\partial p'}{\partial X} = 0, \quad \frac{\partial v_y'}{\partial t} + \frac{1}{\rho}\frac{\partial p'}{\partial Y} = 0$$

Perturbations of density ρ' are eliminated by the assumption of adiabatic conditions

$$\frac{\partial p'}{\partial t} = c^2 \frac{\partial \rho'}{\partial t} \tag{1.2}$$

Conditions at the shock wave front are formulated in the manner presented in [3]

$$v_y' = -U\frac{\partial \xi}{\partial Y}, \quad v_x' = \frac{1+\delta}{2\rho_0 D}p', \quad \frac{\partial \xi}{\partial t} = \frac{1-\delta}{2\rho_0 u}p', \quad X = Vt \tag{1.3}$$

Here $\xi(Y,t)$ is the relative displacement of the shock wave front from the plane $X = Vt$.

Initial conditions are established by the assumption that at $t = 0$, the shock wave front coincides with the interface plane where $v_x' = 0$. It follows from (1.3) that for $t = 0$, $p' = 0$, thus the boundary condition for the pressure is

$$p' = 0 \tag{1.4}$$

The tangential velocity component of v_y' is initially other than zero, and equal to $v_y' = -U\partial\varepsilon / \partial Y$.

Let $\varepsilon(Y) = \Delta \exp(ikY)$, where Δ and k are constants, and for small perturbations

$$k\Delta \ll 1 \tag{1.5}$$

The relationship between all parameters and the coordinate Y is determined by the factor $\exp(ikY)$. By introducing

$$p' / \rho c = w, \quad v_x' = u, \quad v_y' = -iv, \quad kX = x, \quad kct = y \tag{1.6}$$

the problem is reduced to the solution of a system of equations

$$\frac{\partial w}{\partial y}+\frac{\partial u}{\partial x}+v=0, \qquad \frac{\partial u}{\partial y}+\frac{\partial w}{\partial x}=0, \qquad \frac{\partial v}{\partial y}-w=0 \tag{1.7}$$

with boundary conditions

$$w=w_0 f(y) \quad \text{for } x=0, \quad u=Aw, \quad \frac{\partial v}{\partial x}=Bw \quad \text{for } x=\beta y \,(\beta<1) \tag{1.8}$$

Here

$$A=\frac{1+\delta}{2\beta}, \qquad B=\frac{1}{\beta}\left[\frac{1-\delta}{2}\frac{\rho}{\rho_0}-1\right] \tag{1.9}$$

with initial conditions

$$u=0, \quad w=0 \quad v=v_0 \quad \text{for } x=y=0 \quad (v_0=Uk\Delta) \tag{1.10}$$

2. Solution of the boundary problem. Introducing new variables

$$y=r\cosh\theta, \quad x=r\sinh\theta, \quad r=\sqrt{y^2-x^2}, \quad \tan\theta=x/y \tag{2.1}$$

and after certain transformations, the system of equations (1.7) can be presented in the form

$$\frac{\partial w}{\partial r}+\frac{1}{r}\frac{\partial u}{\partial \theta}+v\cosh\theta=0, \qquad \frac{\partial u}{\partial r}+\frac{1}{r}\frac{\partial w}{\partial \theta}+v\sinh\theta=0$$
$$\cosh\theta\frac{\partial v}{\partial r}-\frac{\sinh\theta}{r}\frac{\partial v}{\partial \theta}-w=0 \tag{2.2}$$

where function w satisfies Equation

$$\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}-\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}+w=0 \tag{2.3}$$

Boundary and initial conditions are presented as follows:

$$w=w_0 f(r) \qquad \text{for } \theta=0 \tag{2.4}$$

$$u=Aw, \quad \frac{\partial v}{\partial r}=(B\sinh\theta_0+\cosh\theta_0)\,w \quad \text{for } \theta=\theta_0 \tag{2.5}$$

$$u=0, \quad w=0, \quad v=v_0 \quad \text{for } r=0 \tag{2.6}$$

For the solution of the system of equations (2.2) with boundary and initial conditions as in (2.4) to (2.6), we shall use the Laplace transform with r as variable along the real axis. The transformation functions for boundary and initial conditions are

$$w_1=w_0\int_0^\infty e^{-pr}f(r)\,dr=0, \qquad \theta=0 \tag{2.7}$$

$$u_1=Aw_1, \quad pv_1-v_0=(B\sinh\theta_0+\cosh\theta_0)\,w_1, \quad \theta=\theta_0 \tag{2.8}$$

The system of equations (2.2) after the Laplacian transformation have the form of Equations (2.7) and (2.8) in [4]. Substituting

$$p=\sinh q, \qquad w_1(p,\theta)=\frac{w_2(q,\theta)}{\cosh q} \tag{2.9}$$

a wave equation is obtained for $w_2(q,\theta)$, a general solution of which has the form

$$w_2(q,\theta)=F(q+\theta)+\Phi(q-\theta) \tag{2.10}$$

where F and Φ are arbitrary functions. It will be seen from (2.7) that $\Phi(q) = -F(q)$ for $\theta = 0$, therefore,

$$w_2(q, \theta) = F(q + \theta) - F(q - \theta) \tag{2.11}$$

We shall re-write the second equation of system (2.7) from [4] as follows

$$\frac{\partial}{\partial q}\{pu_1 - [F(q+\theta) + F(q-\theta)] + v_1 \sinh\theta\} = 0$$

Consequently

$$pu_1 - [F(q+\theta) + F(q-\theta)] + v_1 \sinh\theta = \varphi(\theta) \tag{2.12}$$

where $\varphi(\theta)$ is an arbitrary function. It is known from [6] that

$$f(r = 0) = f(0) = \lim pf_1(p, \theta) \quad \text{for} \quad p \to \infty$$

Therefore

$$\begin{gathered} w(0) = \lim_{p\to\infty} pw_1(p, \theta) = 0, \qquad \lim_{q\to\infty} [\tanh q w_2(q, \theta)] = 2F(\infty) = 0 \\ u(0) = \lim_{p\to\infty} pu_1 = 0, \qquad \lim_{q\to\infty} pu_1 = 0 \end{gathered} \tag{2.13}$$

with $\mathrm{Re}\, q \to \infty$ in Equation (2.12), the left-hand side becomes zero for any value of θ, i.e. $\varphi(\theta) = 0$. Taking $\theta = \theta_0$ in (2.12), and noting that $\varphi(\theta_0) = 0$, we find from (2.7) and (2.8) that $F(q)$ satisfies the equation of finite differences

$$\begin{gathered} \sinh 2q\, [F(q+\theta_0) + F(q-\theta_0)] - (a \cosh 2q + b)\, [F(q+\theta_0) - \\ - F(q-\theta_0)] = 2v_0 \sinh\theta_0 \cosh q \end{gathered} \tag{2.14}$$

where

$$a = A = \frac{1+\delta}{2\beta} > 1, \qquad b = 2\sinh\theta_0\, (B \sinh\theta_0 + \cosh\theta_0) - A = \frac{1-\delta}{2\beta}$$

The existence and the uniqueness of solution of a similar equation was proved in [4], while methods for the solution of finite difference equations are given in [7]. We shall seek the solution of (2.14) for the particular case of $\mathrm{Re}\, q \to \infty$ in the form of series

$$F(q) = \sum_{n=0}^{\infty} A_n e^{-(2n+1)q} \tag{2.15}$$

Substituting this expression into (2.14), and equating coefficients at equal exponents, we obtain the following recurrent realtions for the coefficients $C_n = 2A_n \sinh(2n+1)\theta_0$:

$$\begin{gathered} C_0 = \frac{2v_0 \sinh\theta_0}{a + \coth\theta_0}, \qquad C_1 = C_0 \frac{\coth\theta_0 + (a - 2b)}{a + \coth 3\theta_0} \\ [a + \coth(2n+3)\theta_0] C_{n+1} + 2bC_n + [a - \coth(2n-1)\theta_0]\, C_{n-1} = 0 \end{gathered} \tag{2.16}$$

According to (2.11), we have

$$w_2(q, \theta) = -\sum_{n=0}^{\infty} C_n \frac{\sinh(2n+1)\theta}{\sinh(2n+1)\theta_0} e^{-(2n+1)\theta} \tag{2.17}$$

Proof of convergence of this solution is given in the work of Zaidel'[4]. Reverting to the variable $p = \sinh q$ and using the known formula [6] for

Bessel's function
$$J_n(r) \doteq \frac{(\sqrt{p^2+1}-p)^n}{\sqrt{p^2+1}}$$

we obtain after certain transformations
$$w(r,\theta) = -\sum_{n=0}^{\infty} C_n \frac{\sinh(2n+1)\theta}{\sinh(2n+1)\theta_0} J_{2n+1}(r) \tag{2.18}$$

Conversion to initial variables X and ct is arrived at by means of Formulas
$$r = kct\sqrt{1-\tau^2}, \qquad \tau = X/ct$$
$$\cosh(2n+1)\theta = {}^1/_2\,[(1+\tau)^{2n+1} + (1-\tau)^{2n+1}](1-\tau)^{-(n+{}^1/_2)} \tag{2.19}$$

Pressure at the shock wave front is given by the series
$$w(r,\theta_0) = -\sum_{n=0}^{\infty} C_n J_{2n+1}(s), \qquad s = kct\sqrt{1-\beta^2} \tag{2.20}$$

Substituting Expression (2.20) into the second equation for boundary conditions (2.5), we obtain after integration
$$v(r,\theta_0) = v_0 - (B\sinh\theta_0 + \cosh\theta_0)\sum_{n=0}^{\infty} C_n \int_0^s J_{2n+1}(x)\,dx \tag{2.21}$$

We shall note the relationship which follows from Equations (2.14) and (2.17) for $q = 0$
$$w_2(0,\theta_0) = Q = -\sum_{n=0}^{\infty} C_n = -\frac{v_0}{B\sinh\theta_0 + \cosh\theta_0} \tag{2.22}$$

As the values of $v(r,\theta_0)$ and $\xi(s)$ are proportional, the expression for the form of the shock wave front is
$$\frac{\xi(s)}{\Delta} = -\frac{1}{Q}\sum_{n=0}^{\infty} C_n \int_s^{\infty} J_{2n+1}(x)\,dx \tag{2.23}$$

Using Bessel's functions, this result can be presented in the form which does not contain integrals. Denoting
$$G_0(r) = \int_r^{\infty} J_1(x)\,dx, \qquad G_n(r) = \int_r^{\infty} J_{2n+1}(x)\,dx \qquad (n = 1, 2, 3, \ldots) \tag{2.24}$$

Then
$$G_0(r) = J_0(r), \qquad G_n(r) = 2J_n(r) + G_{n-1}(r) \tag{2.25}$$

and we have
$$\sum_{n=0}^{\infty} C_n G_n(r) = C_0 J_0(r) + \sum_{n=1}^{\infty} C_n [2J_{2n}(r) + G_{n-1}(r)] =$$
$$= C_0 J_0(r) + 2\sum_{n=1}^{\infty} C_n J_{2n}(r) + \sum_{n=0}^{\infty} C_{n+1} G_n(r) = C_0 J_0(r) + 2\sum_{n=1}^{\infty} C_n J_{2n}(r) +$$
$$+ C_1 J_0(r) + \sum_{n=1}^{\infty} C_{n+1}[2J_{2n}(r) + G_{n-1}(r)] = (C_0 + C_1) J_0(r) +$$
$$+ 2\sum_{n=1}^{\infty} (C_n + C_{n+1}) J_{2n}(r) + \sum_{n=1}^{\infty} C_{n+1} G_n(r)$$

As $C_n \to 0$ for $n \to \infty$, we obtain, by continuing this process

$$\sum_{n=0}^{\infty} C_n \int_r^{\infty} J_{2n+1}(x)\,dx = J_0(r) \sum_{n=0}^{\infty} C_n + \sum_{n=1}^{\infty} D_n J_{2n}(r) \qquad \left(D_n = 2\sum_{m=n}^{\infty} C_m\right) \quad (2.26)$$

Summation of recurrent equation (2.16) for n to ∞ leads to the following relationship: (2.27)

$$D_n = \frac{1}{a+b}\{[a + \coth(2n+1)\theta_0]\,C_n - [a - \coth(2n-1)\theta_0]\,C_{n-1}\} \quad (n=1,2,\ldots)$$

Thus, the form of the shock wave front is determined, in relation to time, by the series

$$\frac{\xi(s)}{\Delta} = J_0(s) - \frac{1}{Q}\sum_{n=1}^{\infty} D_n J_{2n}(s) \qquad (2.28)$$

3. Certain limit cases. For the asymptotic case with $r \gg 1$ we shall use Bessel's function

$$J_{2n+1}(r) \approx (-1)^n \frac{2}{\sqrt{2\pi r}}\left[\sin\left(r - \frac{\pi}{4}\right) + \frac{4(2n+1)^2 - 1}{8r}\cos\left(r - \frac{\pi}{4}\right)\right] \quad (3.1)$$

We note from the finite difference equation (2.14) that for $q = \frac{1}{2}t\pi$, $a \neq b$, therefore

$$\sum_{n=0}^{\infty}(-1)^n C_n = 0 \qquad (3.2)$$

From (3.1) and (2.18) we find that

$$w(r,\theta) = -\sqrt{\frac{2}{\pi r}}\sin\left(r - \frac{\pi}{r}\right)\sum_{n=0}^{\infty} C_n \frac{\sinh(2n+1)\theta}{\sinh(2n+1)\theta_0} -$$

$$-\frac{\cos(r - {}^1/_4\pi)}{4\sqrt{2\pi r^3}}\sum_{n=0}^{\infty}(-1)^n C_n \frac{\sinh(2n+1)\theta}{\sinh(2n+1)\theta_0}[4(2n+1)^2 - 1] \qquad (3.3)$$

As (3.2) is fulfilled at the front of the shock wave, the first sum of Expression (3.3) becomes zero, and

$$w(r,\theta_0) = -N\frac{\cos(s - {}^1/_4\pi)}{\sqrt{2\pi s^3}} \qquad \left(N = 4\sum_{n=0}^{\infty}(-1)^n n(n+1)C_n\right) \qquad (3.4)$$

a) For the case of a strong shock wave ($c = 0$, or $U \to \infty$) we have $\delta = 0$ and $a = b$. It is to be expected that for this mode, the solution of the asymptotic behavior will be substantially different, specifically, the growth of perturbations will follow a different law. In this case Equation (2.14) becomes (3.5)

$$\sinh q\,[F(q+\theta_0) + F(q-\theta_0)] - a\cosh q\,[F(q+\theta_0) - F(q-\theta_0)] = 2v_0\sinh\theta_0$$

As previously, we shall seek its solution in the form of a series (2.15), and obtain for C_n

$$C_0 = \frac{2v_0\sinh\theta_0}{a + \coth\theta_0}, \quad [a + \coth(2n+1)\theta_0]\,C_n + [a - \coth(2n-1)\theta_0]\,C_{n-1} - 0 \quad (3.6)$$

The convergence of series with such coefficients C_n is evident. The form of the front of a strong shock wave is determined, in relation to time, by Formula (2.28) where D_n follows the relationship

$$D_1 = -C_0 \frac{a - \coth \theta_0}{a}, \qquad D_n = -D_{n-1} \frac{a - \coth (2n-1)\theta_0}{a + \coth (2n+1)\theta_0} \qquad (n = 2, 3, \ldots) \tag{3.7}$$

Asymptotics are determined as previously, and

$$w(r, \theta_0) = M \frac{\sin(s - {}^1/_4\pi)}{\sqrt{2\pi s}} \qquad \left(M = 2iw_2\left(i\frac{\pi}{2}, \theta_0\right) = -2\sum_{n=0}^{\infty}(-1)^n C_n\right) \tag{3.8}$$

b) It is of interest to analize the case of a strong wave for small but finite values of $\delta \ll 1$. By using the integral form of Bessel's function [8], the pressure at the wave front will be represented by

$$w(r, \theta_0) = \frac{2}{\pi} \operatorname{Im} \left[\int_0^{{}^1/_2 i\pi} w_2(q, \theta_0) \sinh(s \sinh q)\, dq \right] \tag{3.9}$$

From (2.14) it follows that

$$w_2(q, \theta_0) = \frac{2\cosh q}{\sinh 2q + a\cosh q + b} \{2 \sinh q F(q + \theta_0) - v_0 \sinh \theta_0\} \tag{3.10}$$

It is evident that the fundamental component of integral (3.9) with $s \gg 1$ is given by point $q = \frac{1}{2}i\pi$, and that the expression in braces in (3.10) can be substituted by its value for $q = \frac{1}{2}i\pi$ and $\delta = 0$. We note that coefficient N in (3.4) for $q = \frac{1}{2}i\pi$ is

$$N = i \frac{\partial^2}{\partial q^2} w_2(q_i \theta_0)$$

after double differentiation, it follows from (3.10) that

$$N = -\frac{4M}{(h+1)\delta^2} \qquad (\text{for } \delta \ll 1) \tag{3.11}$$

In this manner, taking into account (3.11), it follows after necessary transformations of (3.9) and (3.10), that for $s \gg 1$ and $\delta \ll 1$

$$w(r, \theta_0) \approx \frac{2M}{\pi} \int_0^{{}^1/_2\pi} \frac{\sin(s \sin \varphi) \cos \varphi \sin 2\varphi}{(a \cos 2\varphi + b)^2 + (\sin 2\varphi)^2}\, d\varphi$$

Noting that the fundamental term of the integral is determined in the vicinity of point $x = \sin \varphi = 1$, and extending the lower limit of integration to $-\infty$, we obtain

$$w(r, \theta_0) \approx \frac{4\sqrt{2}M}{\pi} \int_{-\infty}^{1} \frac{\sin(sx)\sqrt{1-x}}{(a-b)^2 + 8(1-x)}\, dx \tag{3.12}$$

Making $\alpha = {}^1/_8(h+1)s\delta^2$, we obtain the sought Formula

$$w(r, \theta_0) \sim -\frac{M\sqrt{h+1}}{4\pi} \delta \frac{\partial}{\partial s} \operatorname{Re}\left\{\exp\left[i\left(s - \frac{\pi}{4}\right)\right] \psi(\alpha)\right\}$$

$$\psi(\alpha) = \frac{\sqrt{\pi}}{\sqrt{\alpha}}\left[1 - \sqrt{\pi\alpha}\, e^{i\alpha}\left(e^{{}^1/_4 i\pi} - \frac{2i}{\sqrt{\pi}} \int_0^{\sqrt{\alpha}} e^{-i\eta^2} d\eta\right)\right] \tag{3.13}$$

Substituting this asymptotic expression for $w(r, \theta_0)$ into the second boundary condition (2.5) at the wave front, we have

$$\frac{\xi(s)}{\Delta} \sim -\frac{M}{8\pi v_0}(h+1)\delta\sqrt{h}\,\mathrm{Re}\left\{\exp\left[i\left(s-\frac{\pi}{4}\right)\right]\psi(\alpha)\right\} \qquad (3.14)$$

The asymptotic behavior of function $\Delta^{-1}\xi(s)$ is established by substituting Expressions (3.4) and (3.9) into Formula (2.5) which yields

$$\frac{\xi(s)}{\Delta} \sim \frac{N}{v_0}\frac{1}{2\beta\sinh\theta_0}\frac{\sin(s-{}^1/_4\pi)}{\sqrt{2\pi s^3}} \qquad (\delta \neq 0) \qquad (3.15)$$

In accordance with (3.8), we have for the case of $\delta = 0$

$$\frac{\xi(s)}{\Delta} \sim \frac{M}{2v_0}\sqrt{h(h+1)}\frac{\cos(s-{}^1/_4\pi)}{\sqrt{2\pi s}} \qquad (3.16)$$

A comparison of Formulas (3.15) and (3.16) with Formulas (3.16) in [4] shows that the asymptotic decay of perturbations of the form of the shock wave front in respect to time is, within the accuracy of the constant coefficients, the same for a shock wave moving from a lightly curved piston (see Formula (3.4) in [3]) and for a shock wave penetrating through a lightly curved interface of two media.

c) We shall now consider the case of a weak shock wave ($\delta \to 1$). Here with $\tanh\theta_0 = \beta \to 1$, $\theta_0 \to \infty$. In this case, of all the coefficients, A_n there remains in (2.15) only the coefficient A_0. As for $a \to 1$, $b \to 0$, we have $A_0 = \frac{1}{2}v_0$. It follows from (2.18) that

$$w(r,\theta) = -v_0 J_1(r)\sinh\theta$$

We deduct from this that the formula for perturbations of pressure p' relative to the undisturbed pressure p is

$$\frac{p'}{p} = -\frac{2(h+1)}{h}(M_0-1)k\Delta\exp(ikY)\frac{J_1(kct\sqrt{1-\tau^2})}{\sqrt{1-\tau^2}}\sqrt{4\tau^2-3} \qquad (3.17)$$

The behavior of the front of a weak shock wave $\xi(s)$ for $\delta_0 \to 1$ can be easily established by noting that $F(q+\theta_0) \to 0$ for $\theta_0 \to \infty$. We find from (2.14)

$$w_2(q,\theta_0) \approx -F(q-\theta_0) = -2v_0\sinh\theta_0 e^{-2q}\cosh q \qquad (3.18)$$

As $e^{-q} = \sqrt{p^2+1} - p$, therefore

$$w_1(p,\theta_0) = -2v_0\sinh\theta_0(\sqrt{p^2+1}-p)^2 \qquad (3.19)$$

and by using Laplacian transform tables, we obtain

$$w(r,\theta_0) = -4r^{-1}v_0\sinh\theta_0 J_2(r) \qquad (3.20)$$

Taking into account conditions (2.5) at the wave front, we find from the known function $w(r,\theta_0)$ that

$$v(r,\theta_0) = v_0\left[1-2\int_0^s\frac{J_2(x)}{x}\,dx\right] \qquad \text{for } \theta_0 \gg 1 \qquad (3.21)$$

and using the known Bessel's functions, we dispose of integrals and obtain

$$\frac{\xi(s)}{\Delta} = 2\frac{J_1(s)}{s} \qquad (3.22)$$

which coincides with the respective formula given by Zaidel' in [4]. Therefore, the law of generation of instability in a wave moving from a curved piston, and for a wave penetrating a lightly curved interface of two media, is the same.

4. The law of growth of perturbations of the interface. The penetration of the shock wave through the interface is followed by the movement of the interface itself. At the interface $\theta = 0$ and the second of Equations (2.2) takes the form

$$\frac{\partial u}{\partial r} + \frac{1}{r}\frac{\partial w}{\partial \theta} = 0 \tag{4.1}$$

Substituting into this the expression for $w(r,\theta)$, we obtain

$$\frac{\partial u}{\partial r} = \frac{1}{r}\sum_{n=0}^{\infty} C_n \frac{(2n+1)}{\sinh (2n+1)\theta_0} J_{2n+1}(r) \tag{4.2}$$

We shall use the known Bessel's functions [8] for the integration, and arrive at the expression for the movement of perturbations of the interface

$$u = u_0 + \sum_{n=0}^{\infty} \frac{C_n}{\sinh (2n+1)\theta_0}\left[\int_0^r J_0(x)\,dx + J_{2n+1}(r) - 2\sum_{k=0}^{n} J_{2k+1}(r)\right] \tag{4.3}$$

A second integration yields the expression for the amplitude of distortions of the interface

$$\chi = \chi_0 + \frac{1}{kc}\sum_{n=1}^{\infty} \frac{C_{n-1}}{\sinh (2n-1)\theta_0}\Big[r\int_0^r J_0(x)\,dx - rJ_1(r) - J_0(r) + 2J_{2n}(r) -$$

$$- 2\sum_{k=1}^{n} J_{2k}(r) - 2\sum_{k=1}^{n}\Big(1 - J_0(r) + 2J_{2k}(r) - 2\sum_{l=1}^{k} J_{2l}(r)\Big)\Big] \tag{4.4}$$

We note that for a strong shock wave in a perfect gas with the isentropic index $\gamma = {}^5/_3$, the expression $a - \coth (2n-1)\theta_0 = 0$, for $n = 2$, consequently coefficients C_n and D_n with subscripts 2,3,4 becomes zero. The analytical solution is of the form

$$\frac{\xi(s)}{\Delta} = J_0(s) + \frac{2}{3}J_2(s), \qquad s = kct\sqrt{1-\beta^2} \tag{4.5}$$

$$\frac{\chi}{\chi_0} = -0.1 + 0.9\left[r\int_0^r J_0(x)\,dx - rJ_1(r) + J_0(r)\right]\frac{0.4J_1(r)}{r} \tag{4.6}$$

The asymptotic behavior of the distorted interface for $r \gg 1$ is found easily from Formula (4.4)

$$\chi / \chi_0 \approx r$$

In conclusion, the author wishes to express his sincere gratitude to R.M. Zaidel' for his help, and to L.A. Galin for a useful discussion.

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Translated by J.J.D.